Appendix 1

Coincidence of Equidistant Letter Sequence Pairs in the Book of Genesis

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SUMMARY

Witztum et al. (1994) provide statistical evidence for the non-random coincidence of equidistant letter sequence (*els*) pairs in the Hebrew text of the book of Genesis. Specifically, they show that if an *els* spells the name of a famous Jewish personality, and a second *els* spells the Hebrew date of birth or death of that personality, then the two sequences lie in close proximity to each other more often than expected at random.

We corroborate this unusual result by comparing the proximity measure for each *els* pair with the same measure applied to a probabilistic simulation of the *els* search procedure. We also obtain similar results for a new data set consisting of famous Jewish personality names paired with the Jewish names of the communities in which these personalities were born or died.

KEY WORDS: COMPACTNESS MEASURE, ENCODED INFORMATION, GENESIS, PROXIMITY MEASURE, SIMULATION.

1. INTRODUCTION

This paper describes statistical tests performed to corroborate and extend the highly unusual results reported by Witztum et al. (1994). It is suggested that the reader familiarize himself with this reference before proceeding with this paper.

In their article "Equidistant Letter Sequences in the Book of Genesis", Witztum et al. provide statistical evidence for the non random coincidence of equidistant letter sequence (*els*) pairs in the Hebrew text of the book of Genesis. (The text used is the standard "Textus Receptus" published by the Koren Publishing Company, Jerusalem. Reference Witztum et al. (A.4).) Specifically, they show that if an *els* spells the name of an a priori selected famous Jewish personality and a second *els* spells the Hebrew date of birth or death of that personality then the two sequences can be represented in a mutually compact configuration in which they are in close proximity to each other more often than expected at random. The

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list of personalities selected for this study is referred to as "list 2" by Witztum el al. and is extracted from an encyclopedia of famous rabbinic personalities (Margalioth (ed.), 1961). (See Witztum et al., (A.3) for a detailed description of how this list of personality - date pairs is formed. The criterion for inclusion of a personality in this list is that the entry for the personality in the encyclopedia contain between 1.5 and 3 columns of text and that a date of birth or death be specified. This sample is called "list 2" to distinguish it from an earlier and disjoint sample, "list 1", of personality - date pairs in which the criterion for inclusion was a minimum of 3 columns of text for that entry in the encyclopedia. See Witztum et al., (section 2) for the reason for these two samples.) Witztum et al. compute that the overall significance level for the 163 pairs of names and dates in list 2 found as *els*'s in the book of Genesis is 1.6×10^{-5}. We use a slightly different methodology and corroborate their results. In addition, we produce a new set of equidistant letter sequence pairs, list 3, which pairs the personality names from list 1 and list 2 with the Jewish names of the communities of their birth and death. We show that list 3 exhibits the same phenomenon.

2. EQUIDISTANT LETTER SEQUENCES AND COMPACTNESS

Using the same notation as Witztum et al., we start with an abbreviated description of the methodology and results reported in their paper. The reader is referred to their paper for elaboration of details and motivation.

Define an *els* (equidistant letter sequence), denoted (n, d, k), in the book of Genesis (G) as the sequence of letters found at positions $n, n+d, n+2d, ..., n+(k-1)d$ in G. We call d the "skip distance" of the *els*. Given two *els*'s, $e = (n, d, k)$, $e' = (n', d', k')$ in G, the distance between e and e', $\delta_h(e, e')$, is defined by writing G as a single helix of letters spiraling down a cylinder with h vertical columns of letters and setting $\delta_h(e, e') = f^2 + f'^2 + l^2$, where f is the usual Euclidean distance (in columns or rows of letters) between two consecutive letters of e on the surface of the cylinder, f' is the same for e', and l is the minimal Euclidean distance between a letter of e and one of e' on the surface of the cylinder. Then $\mu_h(e, e') = 1/\delta_h(e, e')$ is directly related to the mutual compactness of the configuration of e and e' (and the proximity of e and e') on the cylinder for given h. That is, the greater the compactness of the configuration of e and e', the larger $\mu_h(e, e')$ tends to be for specified

h. In general, setting $h = h_i$ = the nearest integer to $|d|/i$ tends to give small values of f for i small, so we let h_i = nearest integer to $|d|/i$ and h_i' = nearest integer to $|d'|/i$ and define $\sigma(e,e') = \sum_{i=1}^{10} \mu_{h_i}(e,e') + \sum_{i=1}^{10} \mu_{h_i'}(e,e')$. Note that setting $h = h_i$ as described places the letters of e in a vertical column with distance $f = i$ between successive letters whenever $|d|/i$ is an integer, and places the letters in a non vertical straight line otherwise. See Witztum et al, (A.1). Note too, that $\sigma(e,e')$ tends to be large provided that there is a relatively compact configuration of e and e' and they are in close proximity (i.e., l is small) for at least one of h_i or h_i', $i = 1, ..., 10$.

Suppose the letters of a word W are found as an *els* $e = (n,d,k)$ in G with $|d| \geq 2$. Then T_e is defined as the maximal segment of G such that e is contained in T_e and if W is also found as an *els* $\hat{e} = (\hat{n},\hat{d},k)$ contained in T_e then $|d| \leq |\hat{d}|$. We say e is minimal in T_e. Let $\lambda(T)$ be the length (in letters) of a segment T of G. Then define $w(e,e') = \lambda(T_e \cap T_{e'})/\lambda(G)$. w is a weight, $0 \leq w \leq 1$, which measures the fraction of G in which both e and e' are minimal.

For two words W and W', we now define $\Omega(W,W') = \sum w(e,e')\sigma(e,e')$ where the sum should ideally be taken over all *els*'s e and e' spelling out W and W' respectively. For computational efficiency, however, the sum is taken only over those *els*'s for which $w(e,e')$ is relatively large, and thus contributes substantially to $\Omega(W,W')$. Specifically, let $D(W)$ be the largest skip distance for an *els* e spelling W such that the expected cardinality of $\{e = (n,d,k) | 2 \leq |d| \leq D(W)\}$ is less than or equal to 10. (See Witztum et al., (A.1) for the explicit computation of $D(W)$.) Then the sum in $\Omega(W,W')$ is taken over all *els*'s $e = (n,d,k)$ and $e' = (n',d',k')$ spelling W and W' respectively, such that $2 \leq |d| \leq D(W)$ and $2 \leq |d'| \leq D(W')$. To quote Witztum et al. (pg. 435) "Very roughly, $\Omega(W,W')$ measures the maximum closeness of the more noteworthy appearances of W,W' as *els*'s in Genesis - the closer they are, the larger is $\Omega(W,W')$". "Noteworthy" here means that $w(e,e')$ is relatively large, i.e., the skip distances d and d' of the *els*'s of W and W' respectively are relatively small (compared to other *els*'s of W and W').

It is at this point that we diverge from the methodology of Witztum et al. They define an "(x,y,z) - perturbed *els*", $(n,d,k)^{(x,y,z)}$, where x, y and $z \in \{-2,-1,0,1,2\}$, as the letter

sequence in G at positions $n, n+d, ..., n+(k-4)d, n+(k-3)d+x, n+(k-2)d+x+y$, $n+(k-1)d+x+y+z$. They define $\delta_h((n,d,k)^{(x,y,z)},(n',d',k')^{(x,y,z)})$ in the same way as $\delta_h((n,d,k),(n',d',k'))$ is defined, (and in which f and f' are the Euclidean distances between the unperturbed letters of e and e' respectively) and using the same definition as before, obtain $\Omega^{(x,y,z)}(W,W')$. Note that $\Omega^{(0,0,0)}(W,W') = \Omega(W,W')$.

Let $M(W,W') = \{(x,y,z)|\exists(n,d,k)^{(x,y,z)}$ of W in G and $\exists(n',d',k')^{(x,y,z)}$ of W' in G $\}$ and let $m(W,W') = card(M(W,W'))$. Note that $m(W,W') \leq 125$. If $(0,0,0) \in M(W,W')$ they define $v(W,W') = card(\{(x,y,z) \in M(W,W')|\Omega^{(x,y,z)}(W,W') \geq \Omega(W,W')\})$. If $m(W,W') \geq 10$ then $c(W,W')$ is defined as $v(W,W')/m(W,W')$. (See Witztum et al., (A.2) for more details and motivation.) Note that $c(W,W')$ resembles a normalization of $\Omega(W,W')$: $1/125 \leq c(W,W') \leq 1$. To quote Witztum et al. (pg. 435), "in words, the corrected distance $c(W,W')$ is simply the rank order of the proximity $\Omega(W,W')$ among all the 'perturbed proximities' $\Omega^{(x,y,z)}(W,W')$; we normalize it so that the maximum distance is 1. A large corrected distance means that *els*'s representing W are far away from those representing W', on a scale determined by how far the perturbed *els*'s for W are from those for W'". For technical reasons, Witztum et al. also restrict themselves to *els*'s of words that have between 5 and 8 letters inclusive. (See Witztum et al., (A.3) for the reason.) We do the same here for consistency.

3. PROBABILISTIC SIMULATION OF THE *els* SEARCH

We now deviate from the methodology of Witztum et al. by substituting 124 probabilistic simulations of the *els* search procedure for the 124 perturbations used to normalize $\Omega(W,W')$. Thus, rather than using (x,y,z) - perturbed *els*'s to obtain $c(W,W')$, we instead define $c_s(W,W')$ as a normalized ranking of $\Omega(W,W')$ among $\Omega_s(W,W')$, where each $\Omega_s(W,W')$ is obtained by a probabilistic simulation of the *els* search. That is, let $E(W)$ be the set of *els*'s of a word W in G and let $p(a) = prob(x = a|x \in G)$. Let $a_i, i = 1, ..., k$ be the letters of W and define

$$p(W) \stackrel{def}{=} \prod_{i=1}^{k} p(a_i).$$

The motivation for this definition is that if the a_i's were homogeneously distributed in G and if the skip distance d were large enough to ensure the independence of the $p(a_i)$'s, then

$p(W)$ would be the probability that $(n, d, k) \in E(W)$ for any specified n, d, and k such that $1 \leq n + (k-1)d \leq \lambda(G)$. We stress, however, that no such assumption is made; this is simply the motivation for the definitions and procedure which is to follow.

For specified k and d we now let $M = \lambda(G) - (k-1)|d|$ and define

$$p_d(W, j) \stackrel{def}{=} \binom{M}{j} p(W)^j (1 - p(W))^{M-j}.$$

The motivation here is that if $p(W)$ were $prob((n, d, k) \in E(W)|n, d, k)$, then $p_d(W, j)$ would be the probability of finding j elements of $E(W)$ with skip distance d, i.e.,

$$prob(card(\{x \in E(W)|skip(x) = d\}) = j).$$

Once again, no such assumption is made, this is simply the motivation.

The simulation of the *els* search proceeds as follows. For each d, $2 \leq |d| \leq D(w)$, we produce r_d, a pseudo-random number uniformly distributed on $[0, 1)$, and compare r_d to $p_d(W, j)$ for $j = 1, 2, \ldots, 10$. (We use the program "URAND" on page 246 of Forsythe et al. (1977)). The seed used in the program was computed as $(sec100th + 1)(sec + 1)(min + 1)(hr + 1)$ where $sec100th$, sec, min, and hr are the 1/100th of a second, seconds, minutes, and hour respectively, obtained from the computer clock at the start of execution of the *els* simulation program. The addition of the 1's prevents the product from vanishing.) For each j, if $r_d \leq p_d(W, j)$ we record a simulated *els*, $(\eta_{j,d}, d, k)_s$ of W with $\eta_{j,d}$ as yet unspecified. (Thus, if $r_d \leq p_d(W, j)$ for $j = 1, 2, ..., q$, then q simulated *els*'s are recorded for skip distance d. Recall that the *motivation* of the definition of $p_d(W, j)$ is that under certain conditions, $p_d(W, j)$ *would* be the probability of finding j *els*'s of W at skip distance d.) For the words used in the experiments, the requirement $k \geq 5$ always results in very small values of $p_d(W, j)$ for $j \geq 4$; thus the conservative choice $j \leq 10$. For each $(\eta_{j,d}, d, k)_s$ recorded, we produce another pseudo-random number $r'_{j,d}$, uniform on $[0, 1)$, and set

$$\eta_{j,d} = \begin{cases} [1 + r'_{j,d}(M-1)] & \text{if } d > 0 \\ [1 - (k-1)d + r'_{j,d}(M-1)] & \text{if } d < 0 \end{cases}$$

so that $\eta_{j,d}$ is uniformly distributed over all possible start points of a k long *els* in G with skip distance d. This entire procedure is repeated 124 times to produce sets $E_s(W)$ of simulated

els's of W in G, $s = 1, \ldots, 124$. For each $E_s(W)$ and $E_s(W')$, $s = 1, \ldots, 124$, we compute $\Omega_s(W, W')$ in exactly the same way as $\Omega(W, W')$ is computed, and rank $\Omega(W, W')$ among the $\Omega_s(W, W')$ to produce $c_s(W, W')$, just as $c(W, W')$ is produced by ranking $\Omega(W, W')$ among $\Omega^{(x,y,z)}(W, W')$.

4. THE SIGNIFICANCE LEVEL OF THE COMPACTNESS OF *els* PAIRS

We now return to the approach of Witztum et al. to obtain a probability against the null hypothesis of random correlation between *els*'s of paired words in list 2. Witztum et al. actually define four statistics; we only compute the one (ρ_4) which gave the most significant results. (Note that the final significance level, ρ_0, is a function of the most significant of the four statistics only, viz.: $\rho_0 = 4min(\rho_1, \rho_2, \rho_3, \rho_4)$. Our intention is to corroborate the significance of this final result.) ρ_4 is computed by using a subset, Q, of list 2 in which all appellations starting with the title "Rabbi" are omitted, and taking the product $\Pi(c(W, W'))$ over all word pairs (W, W') in Q. (The use of Q has the effect of reducing the number of personalities with the same title and name in list 2; in fact, all of the personalities in Q have unique appellations. See Witztum et al., (A.5) for a detailed explanation. In the end result, the statistic on Q, ρ_4, as reported in their paper (see Table 3), is only very slightly more significant than ρ_2, the same statistic run on the full list 2 .) P_4 is then defined as $F^N(\Pi(c(W, W')))$ where $N = card(Q)$ and

$$F^N(X) = X(1 - lnX + \frac{(-lnX)^2}{2!} + \cdots + \frac{(-lnX)^{N-1}}{(N-1)!}).$$

Note that if the $c(W, W')$ were independent random variables uniform on $[0, 1]$ then

$$prob(\Pi(c(W, W')) \leq x) = F^N(x).$$

However, no such assumption is made; this is merely the motivation for the definition. See Witztum et al., (A.5) for the details.

To calculate a significance level, 999,999 pseudo-random permutations π_i of the 32 personalities in list 2 are produced, each permutation thus forming a pseudo-random matching of personality name with date of birth or death. Each of these permutations π_i determines a statistic $P_4^{\pi_i}$. Then

$$\rho_4 = \frac{card(\{\pi_i | P_4^{\pi_i} \leq P_4\}) + 1}{10^6}$$

is the probability under the null hypothesis that P_4 would rank as low as it is among the $P_4^{\pi_i}$.

In a similar way, we compute $P_4^{(s)}$ as $F^N(\Pi(c_s(W,W')))$ and $P_4^{(s)\pi}$ as the $P_4^{(s)}$ value computed for a permutation π of the personalities in list 2. We do this computation for 999,999 pseudo-random permutations π_i, where the permutation algorithm and the seed used were the same as in Witztum et al., (A.6), and the pseudo-random number generator was the same as that used for the simulated *els* search. As do Witztum et al., we then compute

$$\varrho_4 = \frac{card(\{\pi_i | P_4^{(s)\pi_i} \leq P_4^{(s)}\}) + 1}{10^6}$$

as the probability under the null hypothesis that $P_4^{(s)}$ would rank as low as it is among the $P_4^{(s)\pi_i}$. Witztum et al. obtain $\varrho_4 = 4 \times 10^{-6}$. (The overall significance level of $1.6 \times 10^{-5} = 4\varrho_4$ is obtained by accounting for the fact that four statistics are computed. See Witztum et al. (section 2). We made an a priori choice to compute only ϱ_4, corresponding to ρ_4, the most significant of the four statistics. Thus, $4\varrho_4$ is an upper bound on an overall significance level based on all four statistics. Recall that the objective is to corroborate the significance of the experiment performed by Witztum et al.)

We also perform a control experiment by repeating the entire procedure using a single letter perturbation in the *els* search. Thus, we compute $c'_s(W,W')$ by calculating $\Omega'(W,W')$ just as $\Omega^{(x,y,z)}(W,W')$ is calculated but using the perturbed *els*'s $(n,d,k)'$ at positions $n, n+d+1, n+2d, n+3d, ..., n+(k-1)d$ (rather than a true *els* at positions $n, n+d, n+2d, ..., n+(k-1)d$) and then ranking $\Omega'(W,W')$ among $\Omega_s(W,W')$ and substituting $c'_s(W,W')$ for $c_s(W,W')$ in all computations to produce ϱ'_4.

5. A NEW EXPERIMENT

The procedures used to calculate ϱ_4 and ϱ_4' were repeated for a new list of word pairs, list 3, and used to produce ϱ_4(list3) and ϱ_4'(list3). No other statistic was computed for list 3. List 3 was formed by using the personality names from list 1 and list 2 paired with the names of the Jewish communities in which the personality was born and in which he died (as opposed to the dates of birth and death). List 3, and the procedure used to construct this list can be found in the appendix which follows.

6. RESULTS AND CONCLUSIONS

The value obtained for ϱ_4 is 7×10^{-6}, supporting the results reported in Witztum et al., (part 3). The intraclass correlation coefficient for the 163 pairs $c(W, W')$, $c_s(W, W')$ for list 2 is 0.8, showing significant correlation between results obtained by the two schemes. (The probability, under the null hypothesis of 0 correlation, of attaining this high an intraclass correlation coefficient on the given sample size can be estimated with Fisher's z transformation: $z = 0.5(\ln(1 + r) - \ln(1 - r))$ is approximately normally distributed with mean $\mu = -0.5\ln(n/(n-1))$ and variance $\sigma^2 = 1/(n - 1.5)$ where r is the intraclass correlation coefficient and n the sample size (reference Fisher (1954)). In our case, with $r = .7961366$, we obtain $(z - \mu)/\sqrt{\sigma^2} = 13.87$ standard deviations which has a right tail probability of 5.14×10^{-44}.) We have also confirmed that the pairings of personalities and dates in list 1 and list 2 are indeed obtained from the referenced encyclopedia. The value obtained for ϱ_4' is 0.435866, well within the range of expectation for a control experiment.

For list 3 we obtain ϱ_4(list3) $= 5 \times 10^{-6}$, supporting the hypothesis that the non random placement of *els*'s in Genesis is not restricted to list 2 (or list 1). Finally, a statistically insignificant result is obtained for the control experiment: ϱ_4'(list3) $= .719061$.

We conclude that these results provide corroboration and extension of the results reported by Witztum, Rips, and Rosenberg. Specifically, the proximity of *els*'s spelling famous Jewish personality names with *els*'s spelling their respective dates of birth and death and communities of birth and death in the Hebrew text of the book of Genesis is very likely not due to chance.

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APPENDIX

We describe the determination of list 3, consisting of all names of personalities from list 1 and list 2 paired with the names of the Jewish communities in which they were born and in which they died.

Introduction

The list of personality names is exactly the same as in list 1 and list 2 . The determination of their places of birth and death is dependent on knowing (a) the place of birth or death, (b) The name of the place, and (c) how to write the name in Hebrew. We use the same encyclopedia (Margalioth (ed.) (1961)) used for the dates of birth and death (ME), but to attain historical and linguistic rigor we compare the data in ME with the data in the "Encyclopaedia Hebraica"(1981) (EH).

A. The places of birth and death

It is easier to know the places of death than the places of birth, since the personalities are not yet famous at the time of birth. Comparing the data in ME and EH we find that: (1) All the places of death given in EH are the same as in ME, (2) For 7 places of birth the EH gives different data than ME, and (3) The EH gives 3 places of birth not mentioned in ME. In all cases we have given precedence to the much more prestigious EH. Note too, that we have been able to verify that for one (number 54 in the list) of the 7 disagreements between ME and EH, the datum in EH was the correct one. It should also be noted that in all cases, the cities of birth given in EH are listed in ME as being relevant to the life of the personality.

B. The names of places

We have two categories: (1) The names of the places as given by the non Jewish residents, and (2) the names given by the Jewish residents to their communities there, which may differ from the names in (1). Our main interest is in the Jewish names of the communities, hence we proceed as follows. For each personality, if ME mentions a name of type (1) which has no parallel of type (2), we use it. If the name is of type (2) or has a parallel of type (2) then we use the name of type (2).

The names of type (2) are well defined in EH and appear there in two ways: (i) In many entries of places, the EH explicitly gives their names in Jewish sources. (ii) Names of places can appear in EH as Jewish family names (e.g., וורמס , i.e., Worms).

C. Hebrew spelling of names

We seek a uniform method of transcription. Since the EH is more rigorous and consistent than ME, we proceed as follows. (a) We start with the index of EH and take the transcription of the name found there. If there are more than one form, we take all of them. (b) If the name is not found in the index, we look for it in the relevant entry in the EH. (c) If it is not mentioned at all in the EH, we copy it from the ME.

The list of names thus obtained still lacks uniformity in some aspects: the use of the "א" as a *mater lectionis*, ending names with "א" or "ה", and how to spell names mentioned in the Torah. We follow the same rules specified in Witztum et al. (A.3). See also Witztum (1989), pg. 72.

D. The names of the Jewish communities

The procedure given in A, B, and C above gives us a set of names. To express the names of the Jewish communities, we use exactly the three forms which are in common (Hebrew) use: (a) The name itself (e.g., וורמס). (b) The name with the prefix קהל (which is also the construct form of the noun קהל - "the community of"), (e.g., קהל וורמס). (c) The name with the prefix קהלת (which is the construct form of קהלה - an equivalent form for

"the community of"), (e.g., קהלת וורמס).

E. The formation of list 3

List 3 consists of a list of personality names paired with the names of the Jewish communities in which the personality was born and in which he died. We adhere strictly to the list of personalities as contained in list 1 and list 2 in Witztum et al., and to the scheme described above for the names of communities. As in the experiment with these personality names and their dates of birth and death, we use only those names consisting of no less than 5 and no more than 8 letters (see Witztum et al. (A.3)). Table 1 contains a list of the personalities and matching city names from which list 3 is formed. Table 2 contains list 3.

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TABLE 1

Personality (from list 1)	Cities
1. The Ra'avad of Posquieres	Narbonne, Posquieres
2. Rabbi Avraham, son of Rambam	Fustat (old Cairo)
3. Rabbi Avraham Ibn Ezra	Tudela
4. Rabbi Eliyahu Bahur	Neustadt, Venice
5. Rabbi Eliyahu of Vilna	Selets, Vilna
6. Rabbi Gershon Ashkenazi	Metz
7. Rabbi David Ganz	Lippstadt, Prague
8. The Taz	Ludomir (Vladimir-Volynski), Lvov (Lwów)
9. Rabbi Haim Ibn-Attar	Salé, Jerusalem
10. Rabbi Yehuda, son of the Rosh	Colonge (Köln), Toledo
11. Rabbi Yehuda Ha-Hasid	Speyer, Regensburg
12. Maharal of Prague	Poznan (Posen), Prague
13. Rabbi Yehonathan Eybeschuetz	Pinczów, Altona
14. Rabbi Heshil of Cracow	Lublin, Cracow (Kraków)
15. The Sema	Lublin, Lvov (Lwów)
16. The Bach	Lublin, Cracow (Kraków)
17. Rabbi Yom-Tov Lipman Heller	Wallerstein, Cracow (Kraków)
18. Rabbenu Yonah	Gerona, Toledo
19. Rabbi Yosef Caro	Safed
20. Rabbi Yehezkel Landa	Opatow, Prague
21. The Pnei-Yehoshua	Cracow (Kraków), Offenbach
22. Rabbenu Tam	-
23. The Rif	Qal'at Hammad, Lucena
24. The Besht	Okop, Medzibezh
25. The Maharam of Rothenberg	Worms, Ensisheim
26. The Levush	Prague, Poznan (Posen)
27. The Rema	Cracow (Kraków)
28. The Ramhal	Padua, Kefar Yasif
29. The Rambam	Córdoba, Fustat (old Cairo)
30. Hacham-Zvi	Lvov (Lwów)
31. The Shach	Holesov
32. Rashi	Troyes
33. The Maharshal	Lublin
34. The Maharsha	Cracow (Kraków), Ostrog

Personality (from list 2)	Cities
35. Rabbi Avraham Av-Beit-Din of Narbonne	Narbonne
36. Rabbi Avraham Yizhaki	Jerusalem
37. Rabbi Avraham Ha-Malakh	Fastov
38. Rabbi Avraham Saba	-
39. Rabbi Aaron of Karlin	Karlin
40. Rabbi Eliezer Ashkenazi	Cracow (Kraków)
41. Rabbi David Oppenheim	Worms, Prague
42. Rabbi David Ha-Nagid	Cairo
43. Rabbi David Nieto	Venice, London
44. Rabbi Haim Abulafia	Hebron, Tiberias
45. Rabbi Haim Benbenest	Istanbul, Izmir
46. Rabbi Haim Capusi	Cairo
47. Rabbi Haim Shabetai	Salonika
48. Rabbi Yair Haim Bacharach	Leipnik, Worms
49. Rabbi Yehudah Hasid	Dubno,Jerusalem
50. Rabbi Yehudah Ayash	Médéa, Jerusalem
51. Rabbi Yehosef Ha-Nagid	Granada
52. Rabbi Yehoshua of Cracow	Vilna, Cracow (Kraków)
53. The Maharit	Safed, Istanbul
54. Rabbi Yosef Tecmim	Steritz (Szczerzec), Frankfurt
55. Rabbi Yakov Beirav	Maqueda, Safed
56. Rabbi Israel Yaakov Hagiz	Fez, Istanbul
57. The Maharil	Mainz, Worms
58. The Yaabez	Altona
59. Rabbi Yizhak Ha-Levi Horowitz	Glogau, Hamburg
60. Rabbi Menahem Mendel Krochmal	Cracow (Kraków)
61. Rabbi Moshe Zacuto	Amsterdam, Mantua
62. Rabbi Moshe Margalith	Kedziniai, Brody
63. Rabbi Azariah Figo	Venice, Rovigo
64. Rabbi Immanuel Hai Ricchi	Ferrara, Reggio
65. Rabbi Shalom Sharabi	San'a, Jerusalem
66. Rabbi Shelomo of Chelm	Zamosc, Salonika

TABLE 2 - List 3

	Jewish Community	Name
1.	נרבונא, נארבונא, פושקיירא	רבי אברהם, הראב"ד
2.	פוסטט, פוסטאט, קהל פוסטט	רבי אברהם
3.	טודלא, קהל טודלא	רבי אברהם, בן עזרא, אבן עזרא, הראב"ע
4.	נוישטאט, נוישטט, ויניציאה, ונציא, קהל ונציא	רבי אליהו, הבחור, בעל הבחור
5.	קהל סלץ, קהלת סלץ, וילנא, קהל וילנא	רבי אליהו, הגאון
6.	קהל מץ, קהלת מץ, קהל מיץ, קהלת מיץ	רבי גרשון, הגרשני
7.	ליפשטט, ליפשטאט, קהל פרג, קהלת פרג, קהל פראג, קהלת פראג	רבי דוד, דוד גנז, דוד גאנז, צמח דוד
8.	לאדמר, קהל לאדמר, קהל לדמר, קהלת לדמר, לודמר, קהל לודמר, קהל לבוב, קהלת לבוב	רבי דוד, דוד הלוי, בעל הט"ז
9.	קהל סאלא, קהלת סאלא, קהל סלא, קהלת סלא, קהל סאלי, קהלת סאלי, קהל סלי, קהלת סלי, ירושלם	רבי חיים, בן עטר, אבן עטר, אור החיים
10.	קולוניא, טולידו, טוליטולא	רבי יהודה
11.	שפירא, קהל שפירא, אשפירא, שפייער, שפייר, רגנסבורג	רבי יהודה
12.	פוזנא, קהל פוזנא, קהל פרג, קהלת פרג, קהל פראג, קהלת פראג	רבי יהודה, רבי ליוא, המהר"ל, מהר"ל מפרג
13.	פינצוב, אלטונא, קהל אה"ו, קהלת אה"ו	רבי יונתן, איבשיץ, בעל התמים
14.	לובלין, קראקא, קהל קראקא, קהל קרקא, קהלת קרקא	רבי יהושע, רבי העשיל
15.	לובלין, קהל לבוב, קהלת לבוב	רבי יהושע, בעל הסמ"ע
16.	לובלין, קראקא, קהל קראקא, קהל קרקא, קהלת קרקא	רבי יואל, סירקש, בעל הב"ח
17.	ולרשטיין, קראקא, קהל קראקא, קהל קרקא, קהלת קרקא	
18.	גירונא, גירונדא, ירונא, קהל ירונא, טולידו, טוליטולא	רבי יונה, רבנו יונה
19.	קהל צפת, קהלת צפת	רבי יוסף, יוסף קרו, יוסף קארו, מהר"י קרו, מיהר"י קארו, בית יוסף, המחבר
20.	קהל אפטא, קהלת אפטא, קהל פרג, קהלת פרג, קהל פראג, קהלת פראג	בעל הצל"ח
21.	קראקא, קהל קראקא, קהל קרקא, קהלת קרקא, אופיבאך	פני יהושע
22.		רבי יעקב, רבנו תם
23.	קלעת חמאד, קלעת חמד, אליסנא, אליסאנא, לוזינא	רבי יצחק, אלפסי, רב אלפס
24.	אוקוף, קהל אוקוף, מזיבוז	רבי ישראל, בעל שם טוב, הבעש"ט
25.	וורמס, קהל וורמס, ורמיזא, וורמשא, וירמיישא, וירמיזא, גרמיזא, אנזיסהים	רבי מאיר, המהר"ם
26.	קהל פרג, קהלת פרג, קהל פראג, קהלת פראג, קהל פוזן, קהלת פוזן	רבי מרדכי, מרדכי יפה, הלבוש, בעל הלבוש
27.	קראקא, קהל קראקא, קהל קרקא, קהלת קרקא	רבי משה, איסרלש
28.	פאדוא, קהל פאדוא, קהל פדוא, קהלת פדוא	לוצטו, לוצאטו, הרמח"ל
29.	קורדובא, פוסטט, קהל פוסטט, פוסטאט	רבי משה, הרמב"ם
30.	קהל לבוב, קהלת לבוב	רבי צבי, חכם צבי
31.	הלישוי	רבי שבתי, שבתי כהן, שבתי הכהן, בעל הש"ך
32.	טרוייש	רבי שלמה
33.	לובלין	רבי שלמה, לוריא, מהרש"ל, המהרש"ל
34.	קראקא, קהל קראקא, קהל קרקא, קהלת קרקא, אוסטרהא	אידלש, מהרש"א, המהרש"א

	Name	
Jewish Community		
נרבונא, נארבונא	רבי אברהם, הראב"י, הרב אב"ד הראב"ד, האשכול	35.
ירושלם	רבי אברהם, יצחקי, זרע אברהם	36.
פסטוב, קהל פסטוב, פאסטוב	רבי אברהם, המלאך	37.
	רבי אברהם, אברהם סבע, צרור המר	38.
קרלין, קהל קרלין, קארלין	רבי אהרן	39.
קראקא, קהל קראקא, קהל קרקא, קהלת קרקא	מעשי השם, מעשי י/ה/ו/ה	40.
וורמס, קהל וורמס, ורמיזא, וורמשא, וירמיישא, וירמיזא, גרמיזא, קהל פרג, קהלת פרג, קהל פראג, קהלת פראג	רבי דוד, אופנהים	41.
קאהיר, קהל קאהיר, קהל קהיר, קהלת קהיר	רבי דוד, דוד הנגיד	42.
ויניציאה, ונציא, קהל ונציא, לונדון	רבי דוד, דוד ניטו	43.
חברון, קהל חברון, טבריא, קהל טבריא	רבי חיים	44.
קושטא, קהל קושטא, אזמיר, קהל אזמיר	רבי חיים, בנבנשת	45.
קאהיר, קהל קאהיר, קהל קהיר, קהלת קהיר	רבי חיים, כפוסי, בעל נס, בעל הנס	46.
שלוניקי, שאלוניקי	רבי חיים, חיים שבתי, מהרח"ש, המהרח"ש	47.
לייפניק, וורמס, קהל וורמס, ורמיזא, וורמשא, וירמיישא, וירמיזא, גרמיזא	תוח יאיר	48.
דובנא, קהל דובנא, ירושלם	רבי יהודה	49.
אלמדיא, ירושלם	רבי יהודה, מהר"י עיאש	50.
גרנדא, קהל גרנדא, גראנאדא	רבי יהוסף	51.
וילנא, קהל וילנא, קראקא, קהל קראקא קהל קרקא, קהלת קרקא	רבי יהושע, מגני שלמה	52.
קהל צפת, קהלת צפת, קושטא, קהל קושטא	רבי יוסף, מטרני, יוסף טרני, טראני מטראני, מהרימ"ט, המהרימ"ט, מהרי"ט, המהרי"ט	53.
סטריץ, קהל סטריץ, פרנקפורט	רבי יוסף, תאומים, פרי מגדים	54.
קהל מקדא, מאקדא, קהל מאקדא, קהלת מקדא, קהל צפת, קהלת צפת	רבי יעקב, יעקב בירב, מהר"י בירב, הריב"ר	55.
קהל פס, קהלת פס, קהל פאס, קהלת פאס, קושטא, קהל קושטא	חאגיז, בעל הלק"ט	56.
מגנצא, קהל מגנצא, מאגנצא, קהל מינץ	רבי יעקב, מולין, יעקב סג"ל, יעקב הלוי, מהר"י סג"ל, מהר"י הלוי, מהרי"ל, המהרי"ל	57.
קהלת מינץ, מאינץ, קהל מאינץ, וורמס, קהל וורמס, ורמיזא, וורמשא, וירמיישא, וירמיזא, גרמיזא		
אלטונא, קהל אה"ו קהלת אה"ו	היעב"ץ, הריעב"ץ, עמדין, הר"י עמדן הר"י עמדי	58.
גלוגא, קהל גלוגא, המבורג, האמבורג, קהל אה"ו, קהלת אה"ו	רבי יצחק, הורוויץ, יצחק הלוי	59.
קראקא, קהל קראקא, קהל קרקא, קהלת קרקא	רבי מנחם, קרוכמל, רבי מענדל, צמח צדק	60.
אמשטרדם, אמשטרדאם, מנטובא, מאנטובא	רבי משה, זכותא, זכותנו, משה זכות, משה זכותא, משה זכותנו, מהר"ם זכות, מהרמ"ז, המהרמ"ז, המזל"ן, קול הרמ"ז	61.
קיידן, קהל קיידן, קיידאן, ברודי, קהל ברודי	רבי משה, מרגליות, פני משה	62.
ויניציאה, ונציא, קהל ונציא, ווויגו	רבי עזריה	63.
קהל פרארא, קהלת פרארא, פראראא, קהל פראראא, קהלת רגו	א"ח הע"ר, ישר לבב	64.
קהל צנעא, קהלת צנעא, ירושלם	רבי שלום, מזרחי, שרעבי, שר שלום, מהרש"ש, המהרש"ש	65.
זמושץ, קהל זמושץ, זאמושץ, שלוניקי, שאלוניקי	רבי שלמה	66.